



**MINDSHARE**  
*2023*

September  
19th - 20th  
2023

# Post-Quantum Tutorial

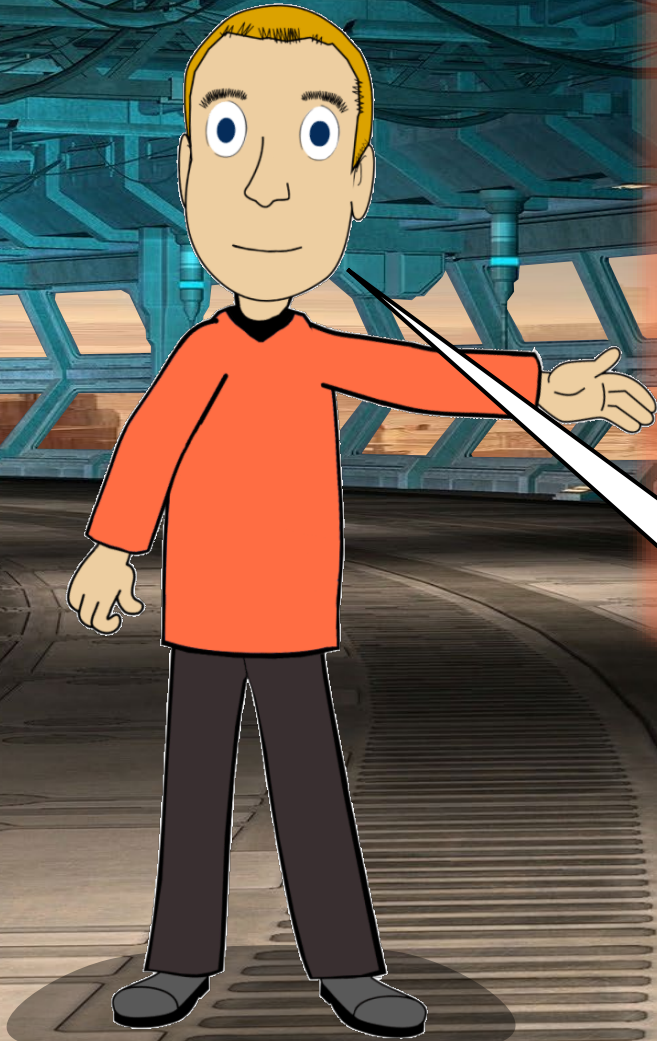
Klaus Schmeh  
Eviden

Let's play  
a quiz.

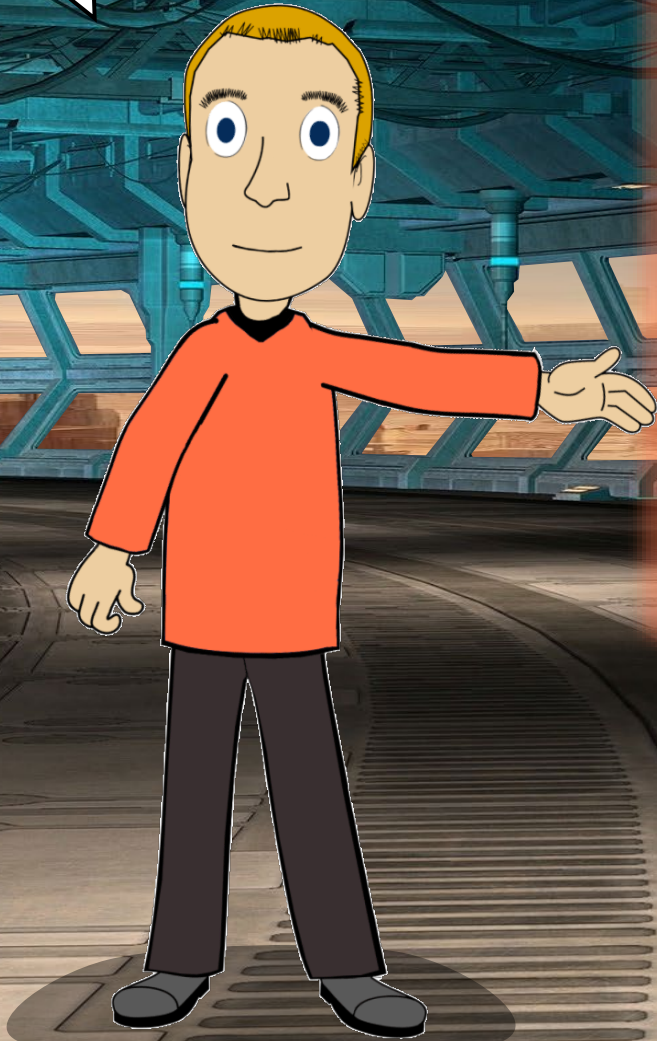
$$31 \times 29 = ?$$

$$31 \times 29 = 899$$

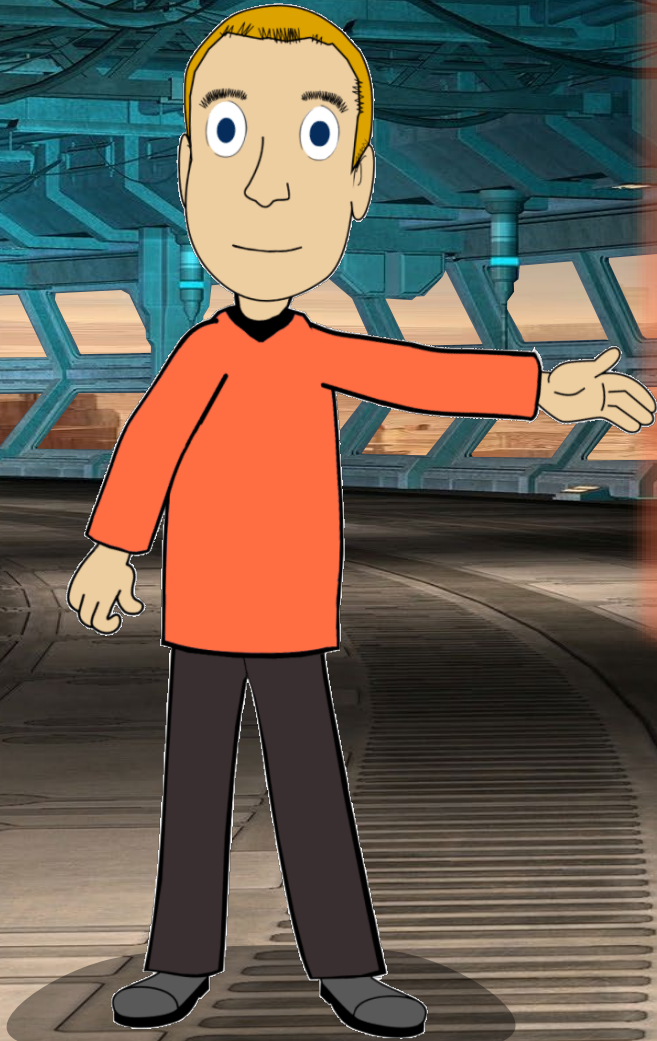
**Prime  
numbers**



Another  
quiz!



$$? \times ? = 851$$



$$37 \times 23 = 851$$

**Multiplying two prime numbers is much easier than the reverse operation.**



**A third  
quiz!**




? \* ? = 1230 186684530 11775513049  
4958384962720772853569595334792  
1973224521517264005072636575187  
4520219978646938995647494277406  
3845925192557326303453731548268  
5079170261221429134616704292143  
1160222124047927473779408066535  
14 19597459856902143413

**Current world record  
in factorizing: 232  
digits or 768 bits**

334780716989568987860441698482126908177  
047949837137685689124313889828837938780  
7614711652531743087737814467999489  
0436667995904282446337996279526322  
64343087642676032283815739666511279  
373417143396810270092798736308917

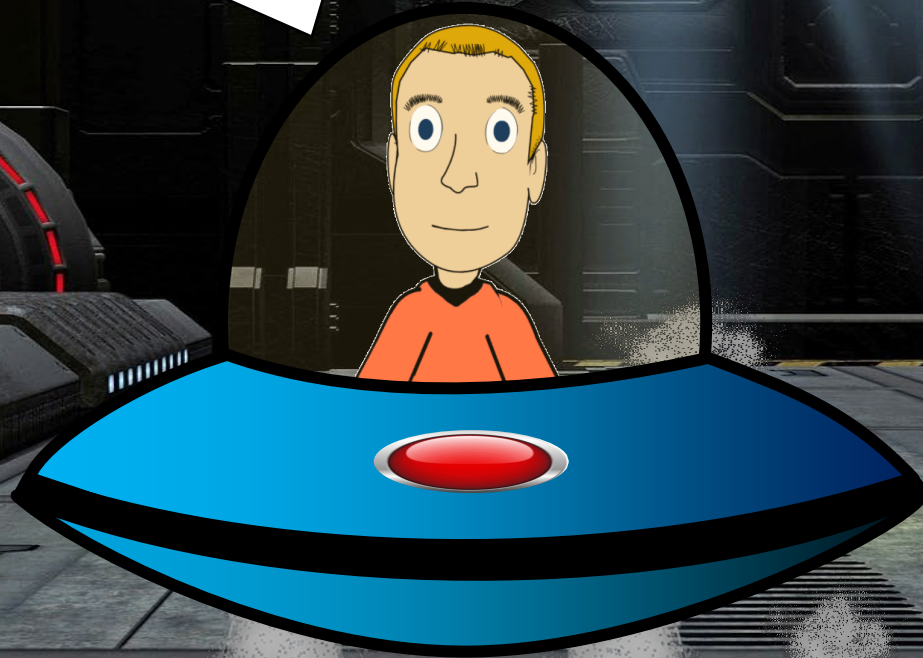
Multiplying prime numbers is easier than factorizing.



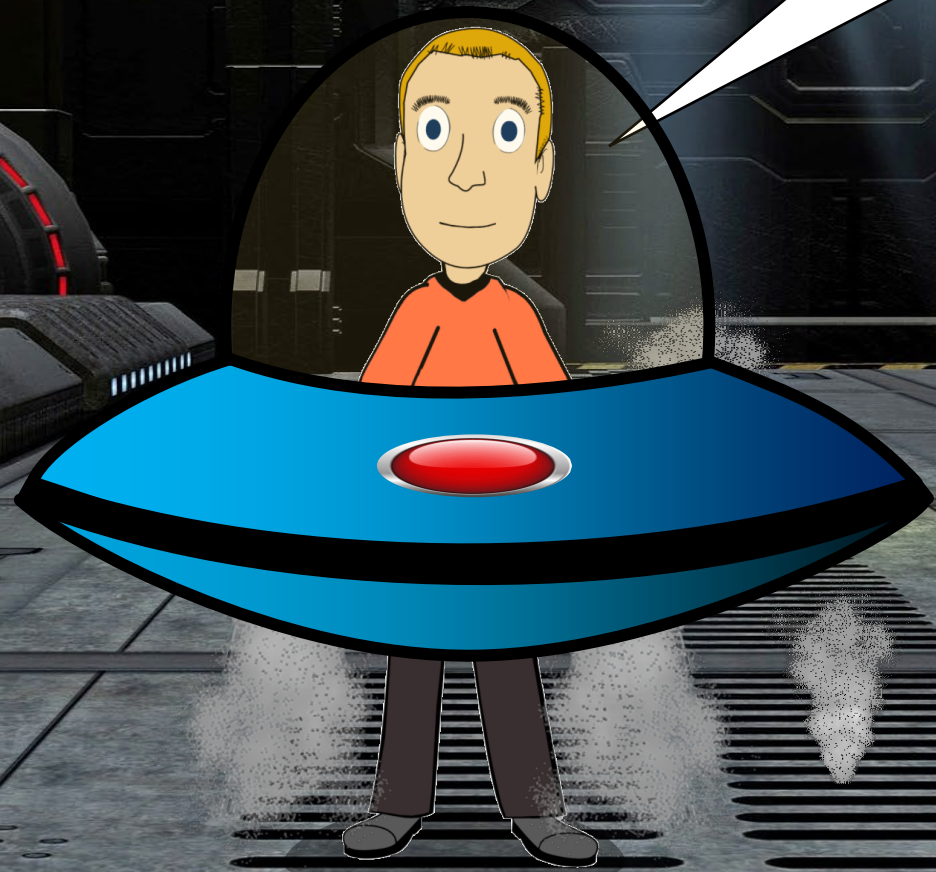
**Even the best computer needs billions of years to factorize a 2048-bit number.**



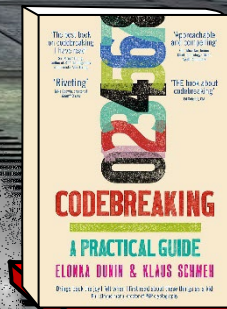
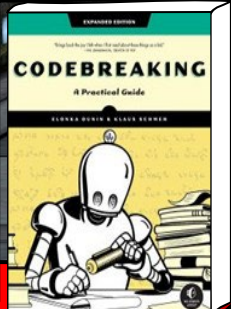
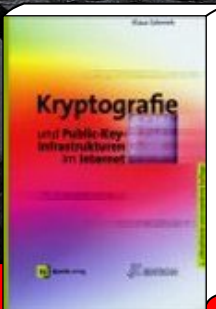
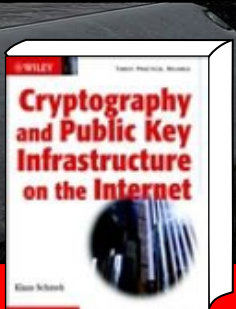
**Klaus Schmeh, Marketing  
Editor at Eviden.**



**Book author,  
blogger**



My  
books

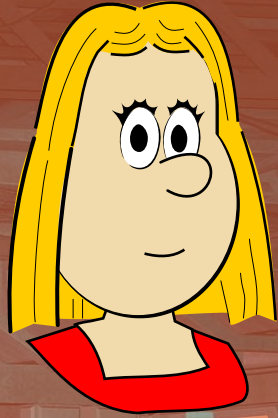


Multiplying prime numbers is easier than factorizing.



**This concept is used for the RSA crypto system.**

# RSA Encryption



Alice

$$17 \cdot 23 = 391$$

Alice's private key  
for decryption

Alice's public key for  
encryption

Public key can easily be computed from private key, but not the other way around

Public key factorized  $\Rightarrow$  encryption is broken

In practice, public key has length of 700 digits

**RSA is used almost everywhere.**



**Not only for encryption, but also for signing.**

## Where is RSA used?



Smartphone



Web browser



Operating system



ATM

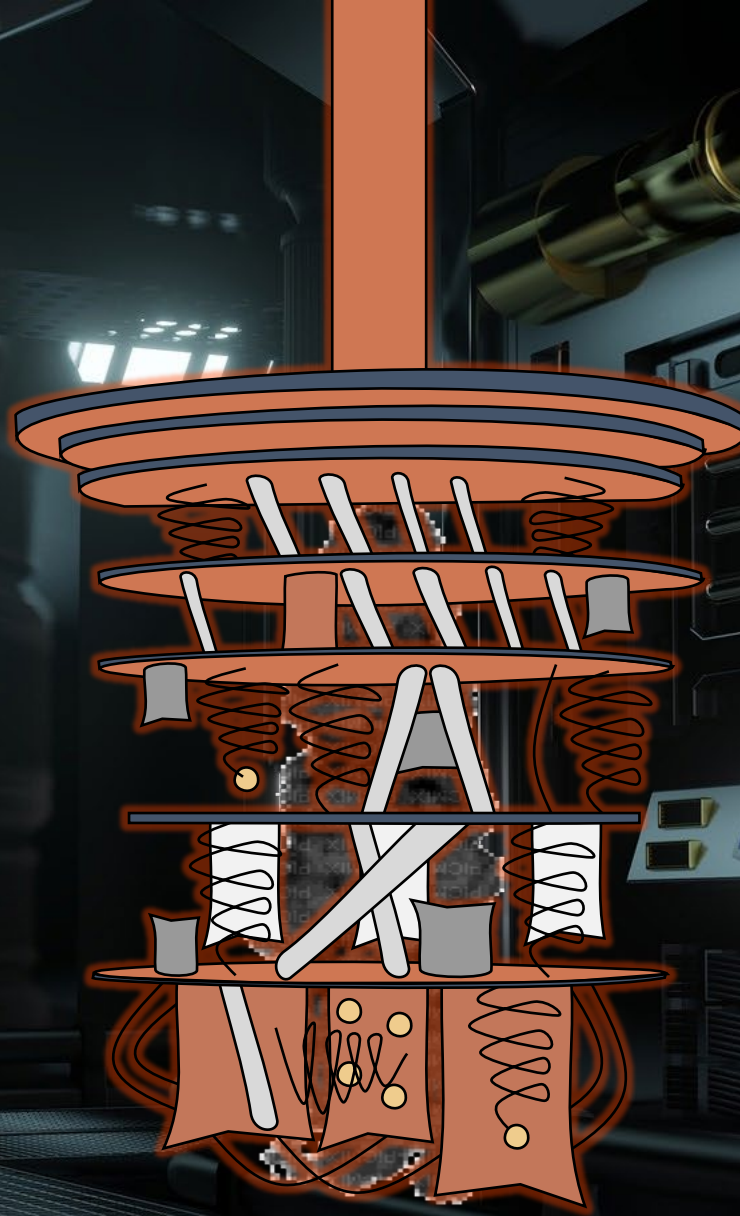
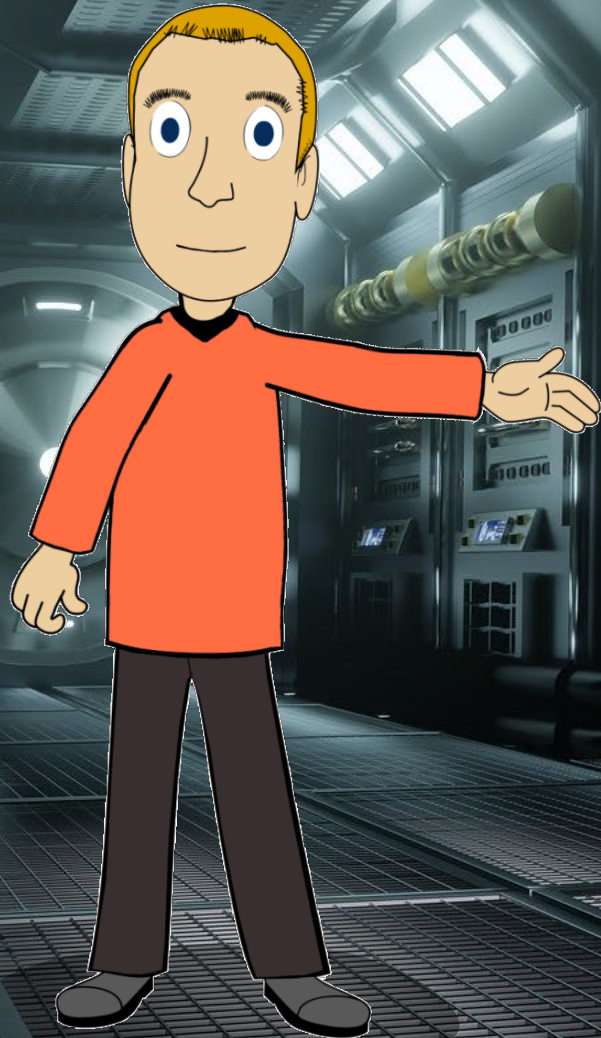


Email client

**This is a quantum computer.**

**Hello!**

**I use quantum mechanics for my work.**

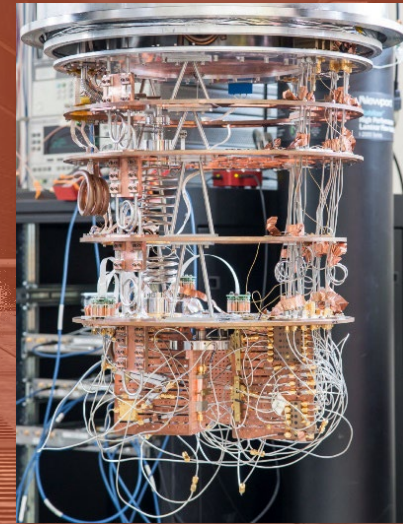


# Schrödinger's cat



Simultaneously  
dead and alive  
Until you look at it

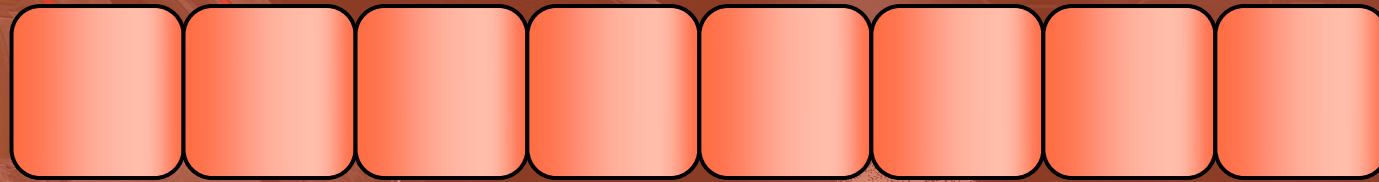
# Quantum bit



Simultaneously  
0 and 1  
Until you read it



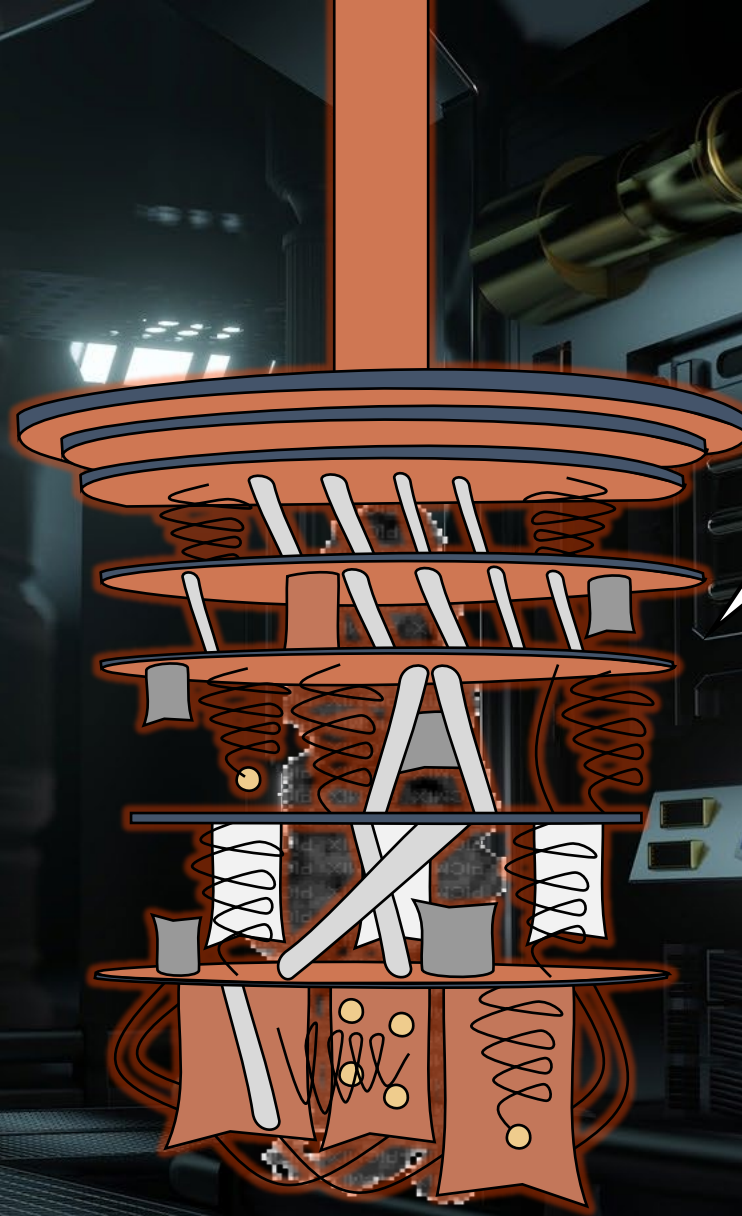
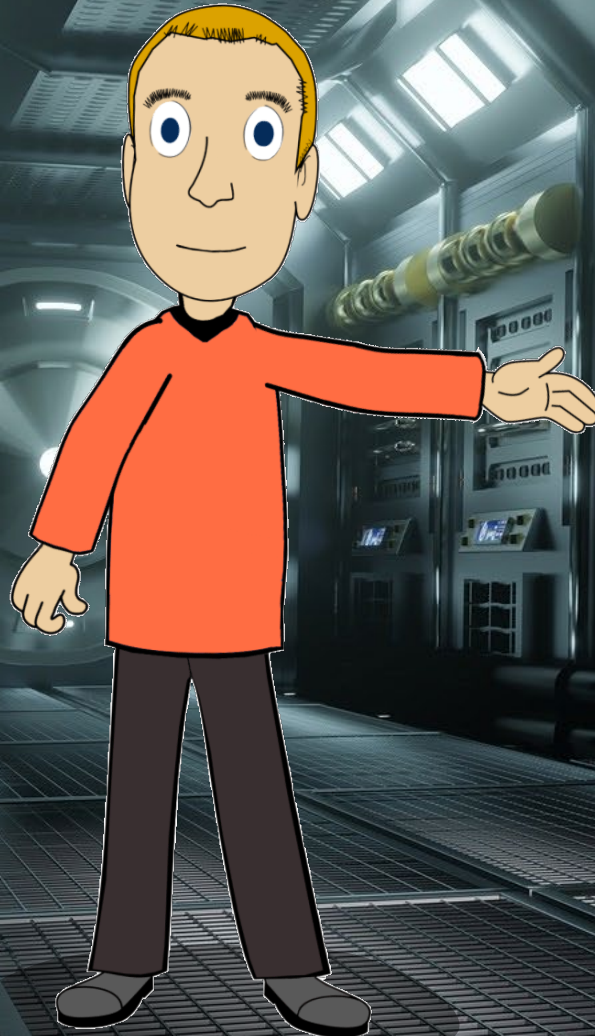
8 quantum bits



256 computations in  
parallel, only one result

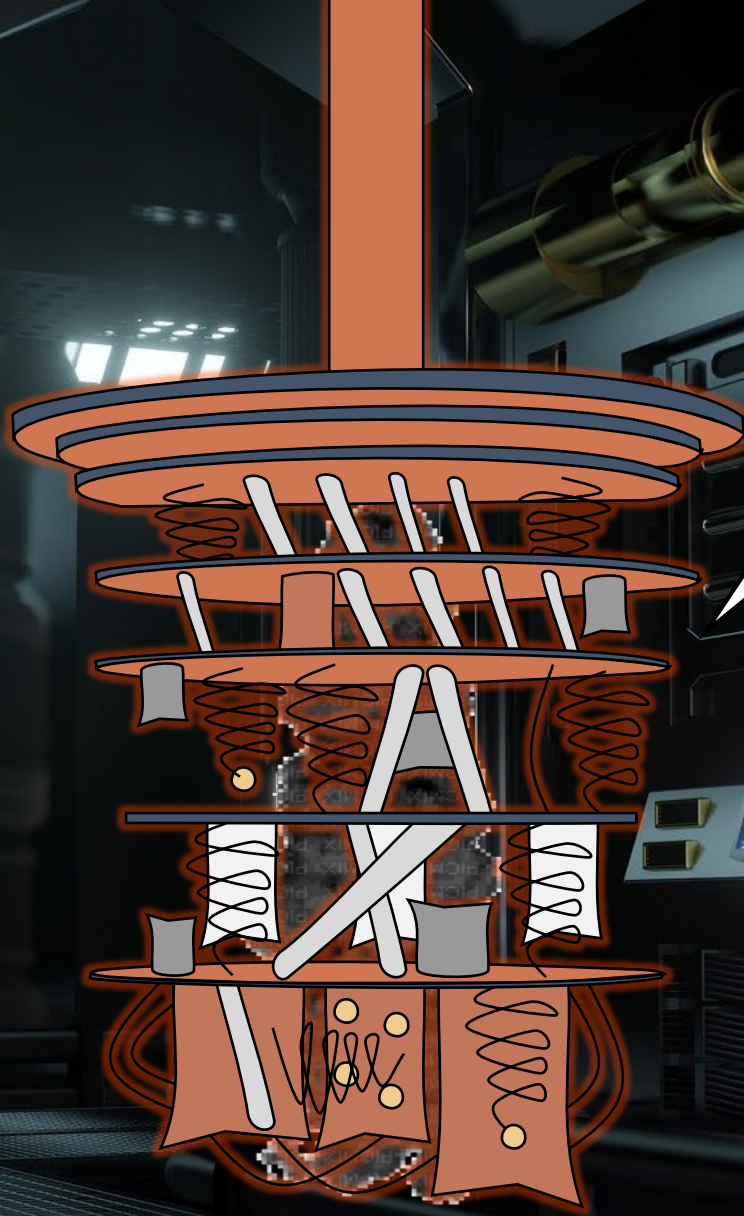
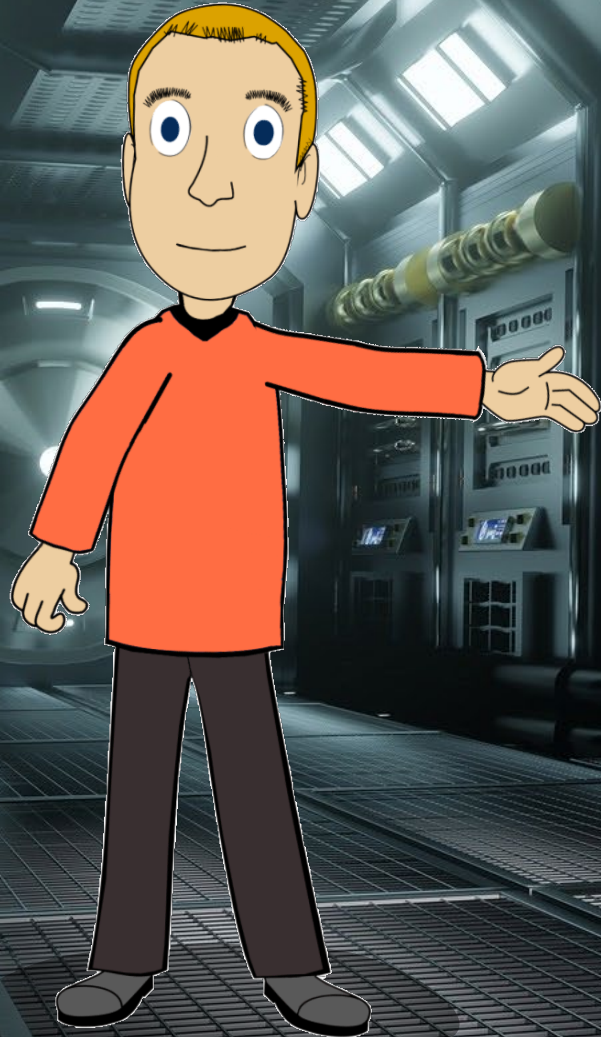
**What are you  
good at?**

**Search huge  
databases, find  
optimal  
solutions.**



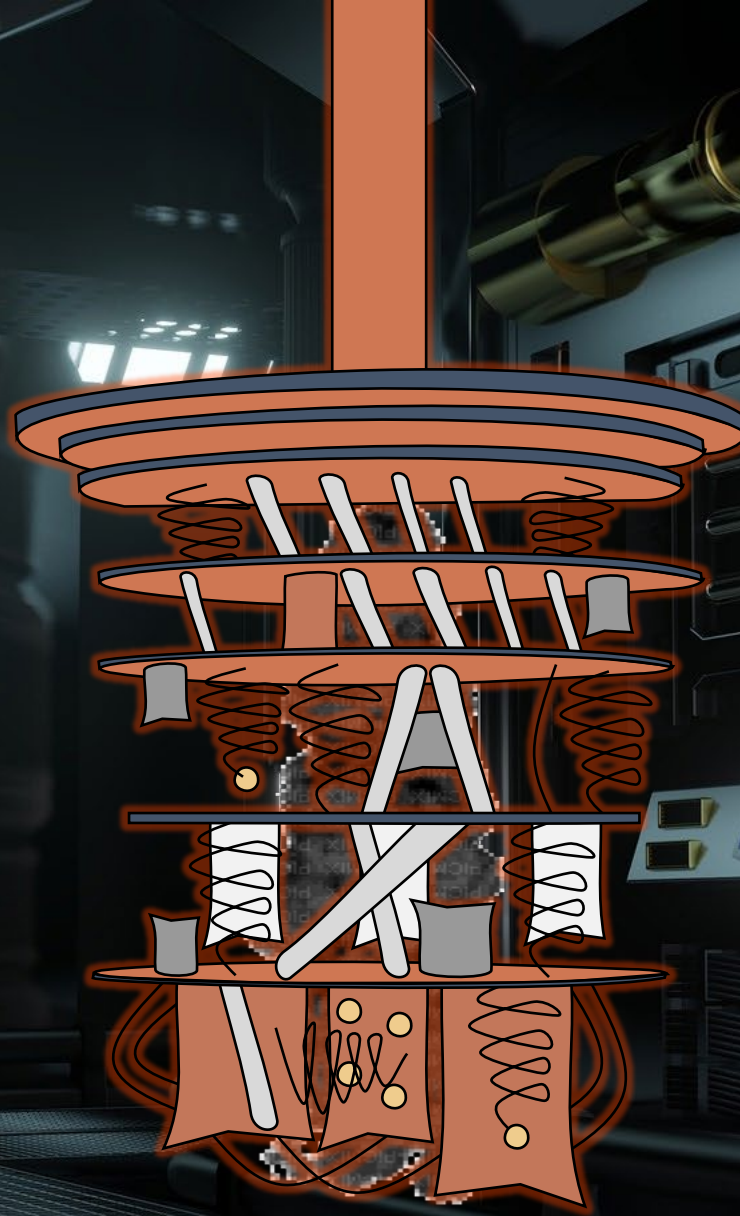
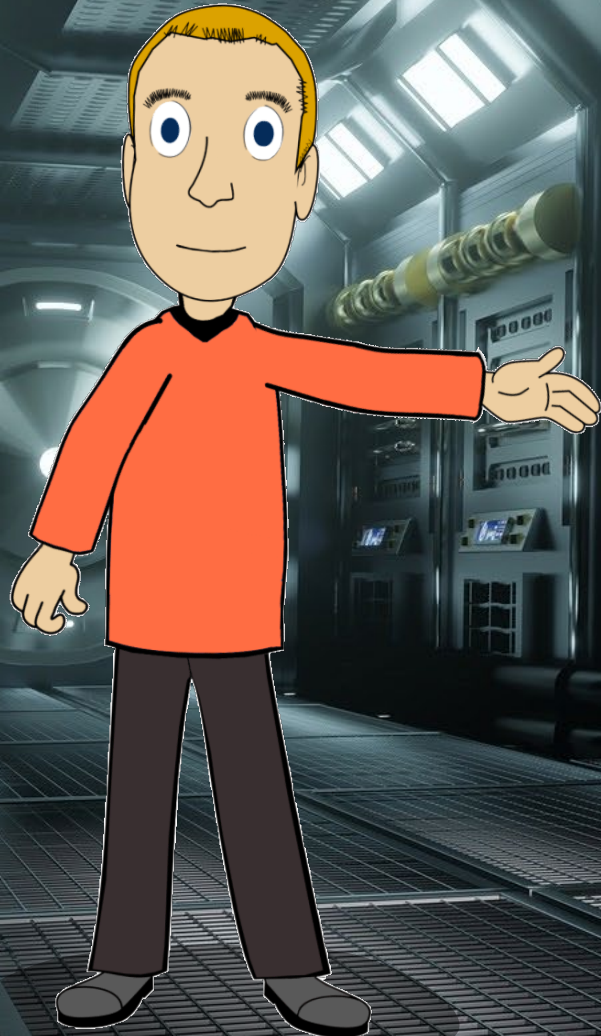
**What are you  
not good at?**

**Tasks that do not  
involve a single  
solution, such as  
sorting.**



What are you particularly good at?

Factorize prime products!





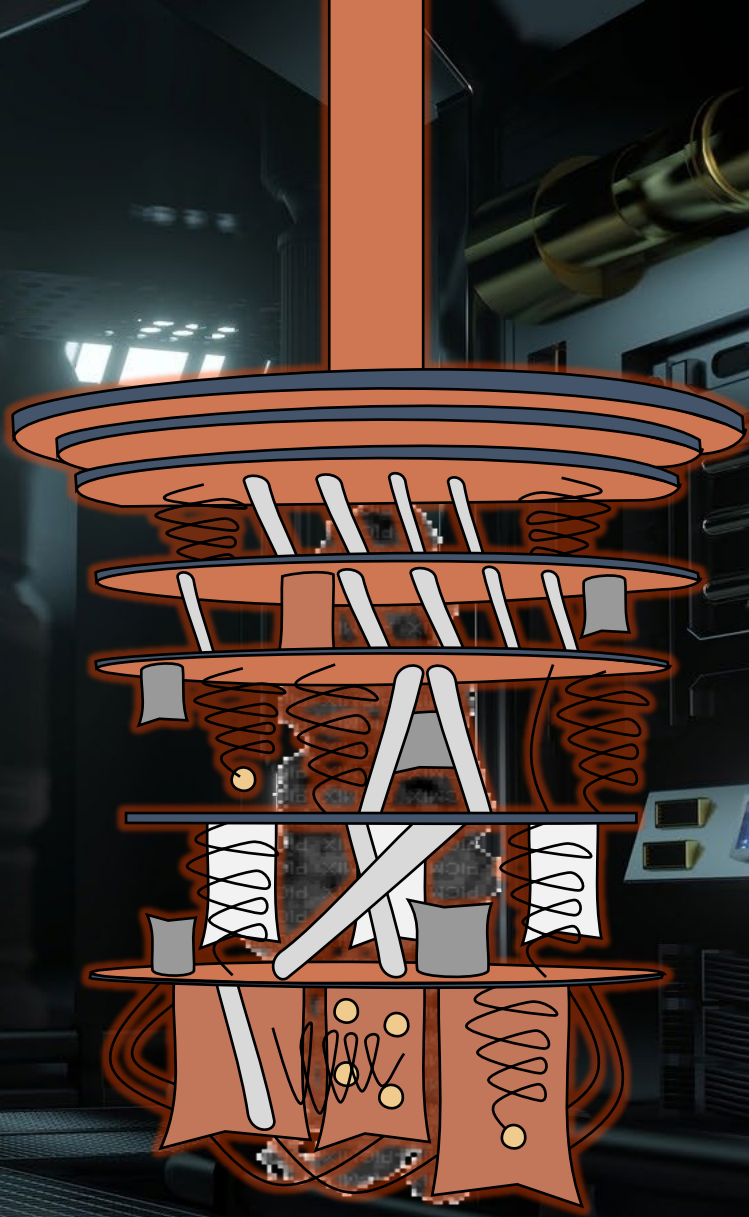
**Factorize prime products?**

**Yes, up to a key length of 5 bits.**

**Can you break RSA?**

**In the future, there could be more.**

Well then!



# Where is RSA used?



Smartphone



Web browser



Operating system

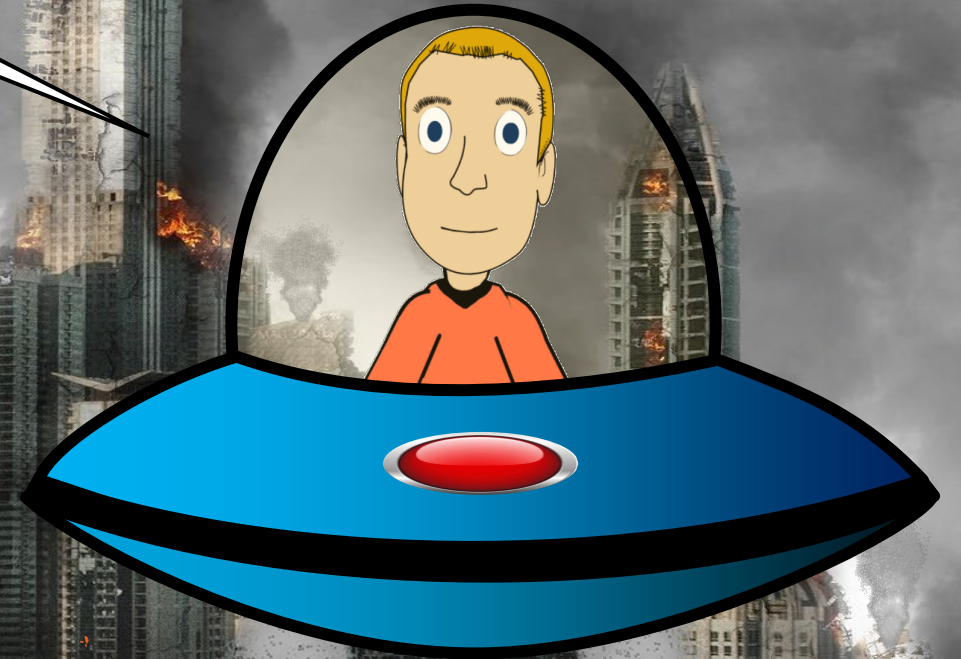


ATM



Email client

**A disaster  
threatens!**

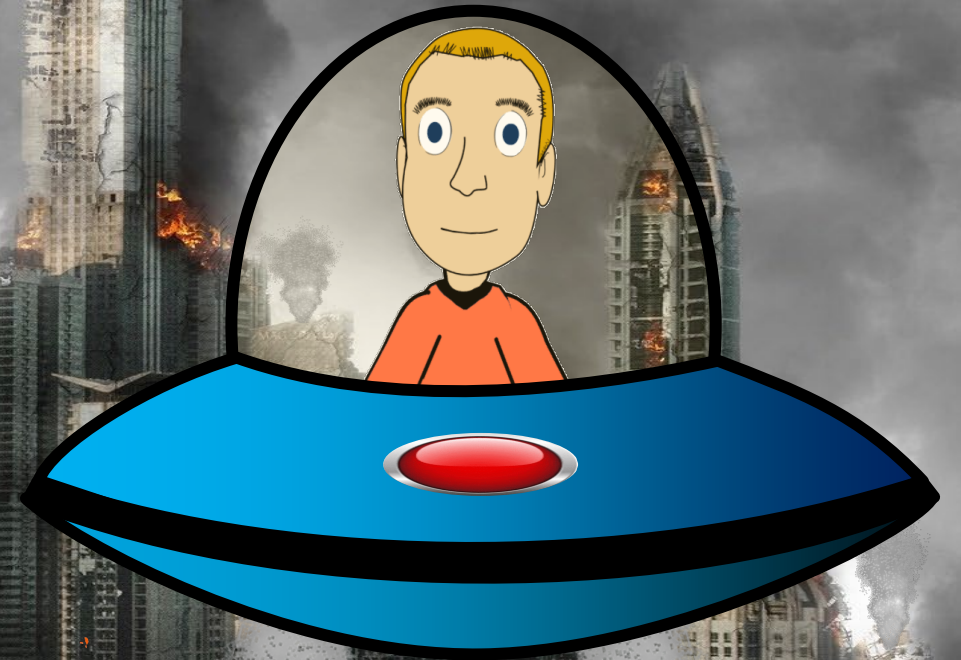




# Q-Day and Y2Q

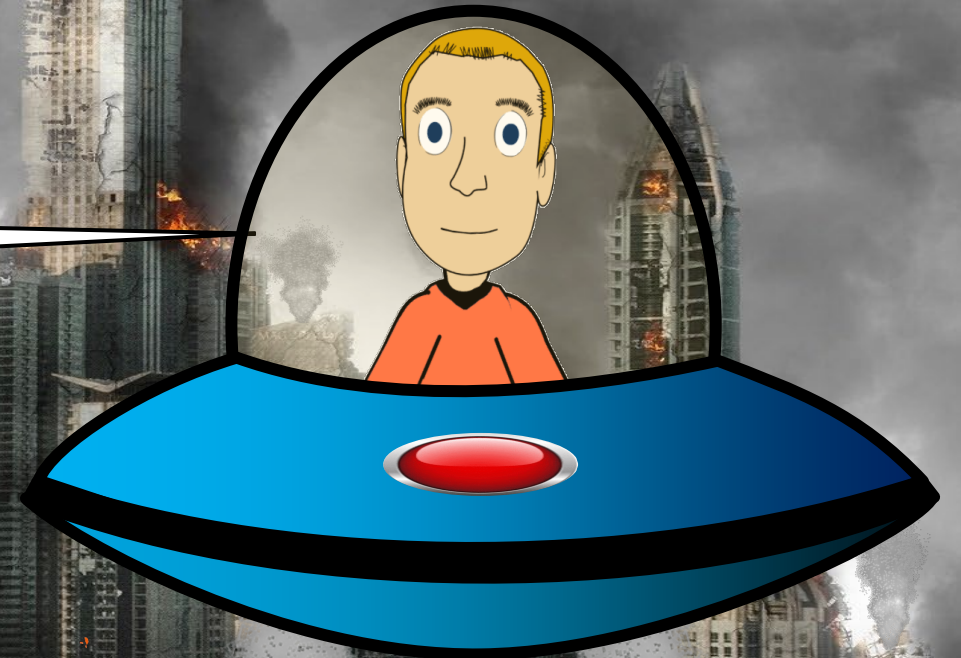
Q-Day: Day from which  
powerful quantum  
computers exist

Y2Q: Year of the Q-Day



**We need  
alternatives to RSA.**

**We need Post-Quantum  
Cryptography.**



# Post-Quantum Cryptography

Crypto methods that cannot be  
broken by a quantum computer

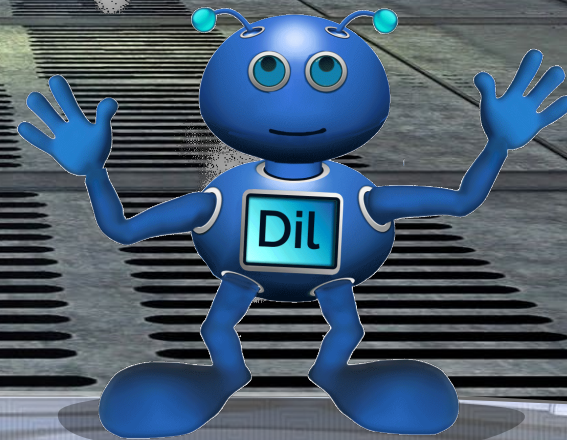
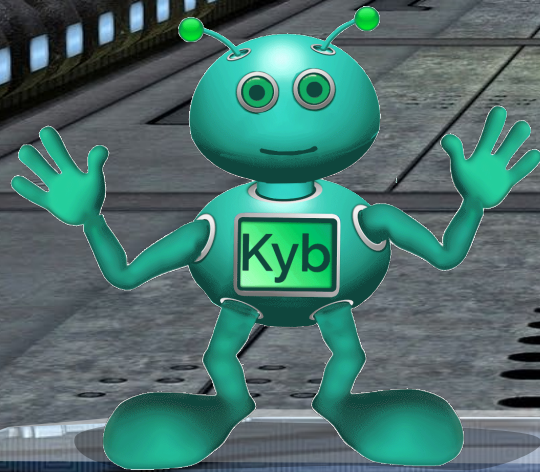
2022

An important year for post-  
quantum cryptography

Because standardization  
has made progress

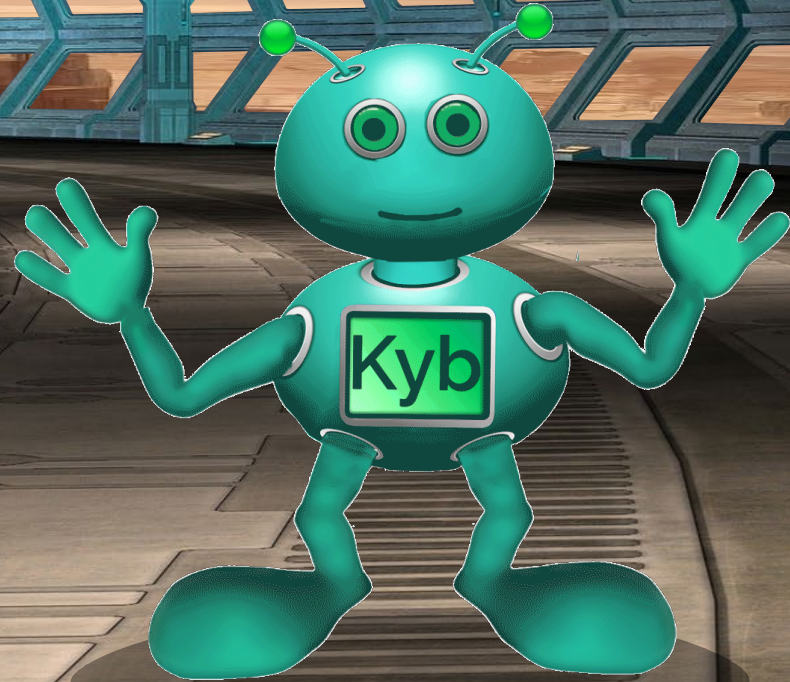
U.S. agency NIST names four post-quantum algorithms as competition winners

Among them CRYSTALS-Kyber and CRYSTALS-Dilithium

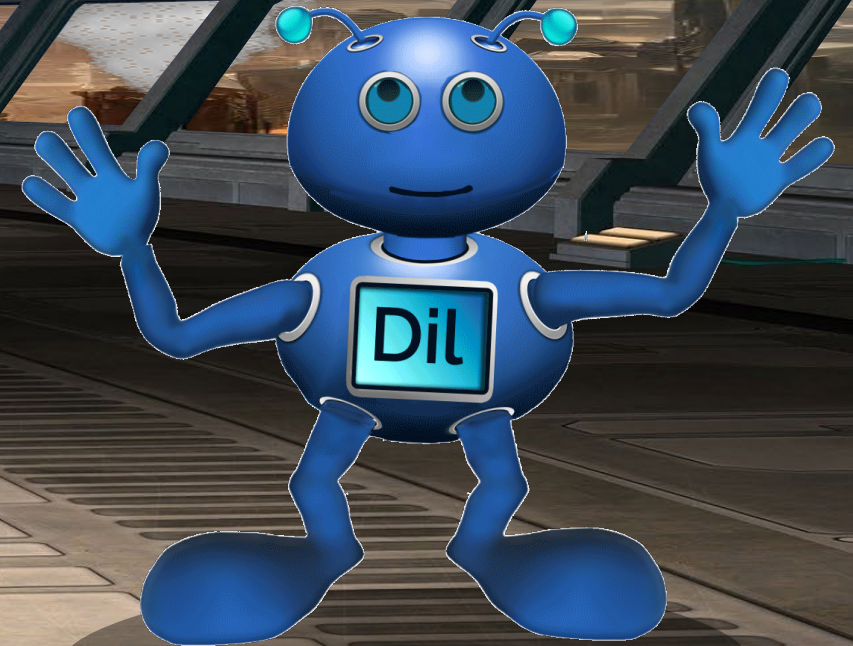


Asymmetric encryption  
method, replacement  
for RSA encryption

Kyber



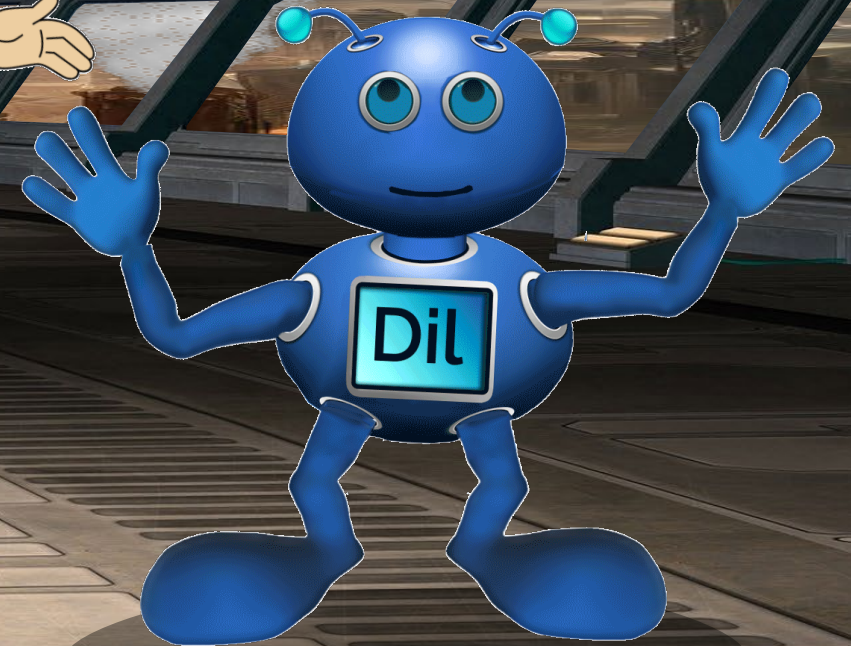
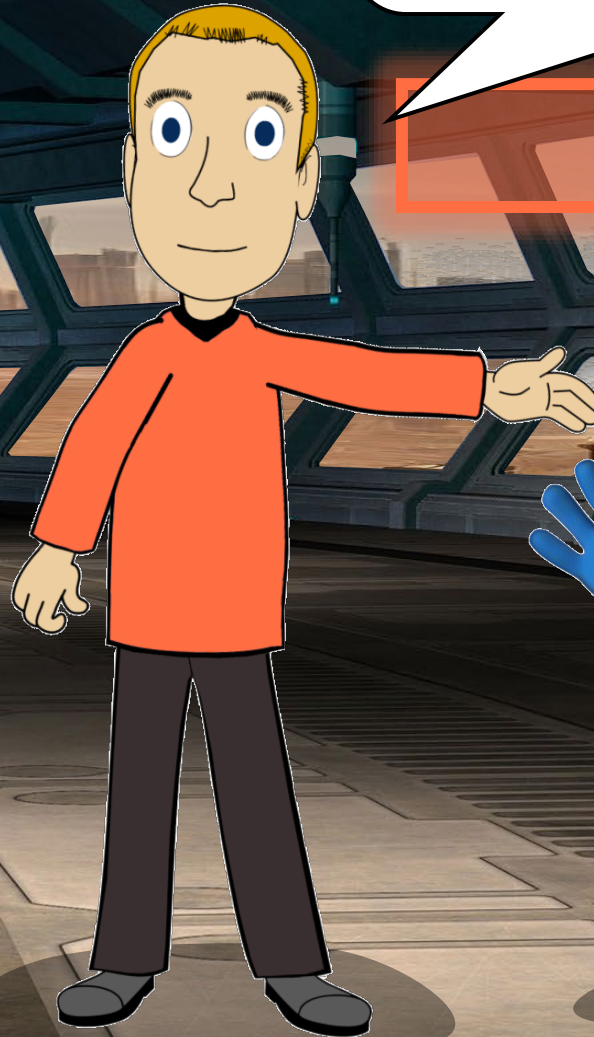
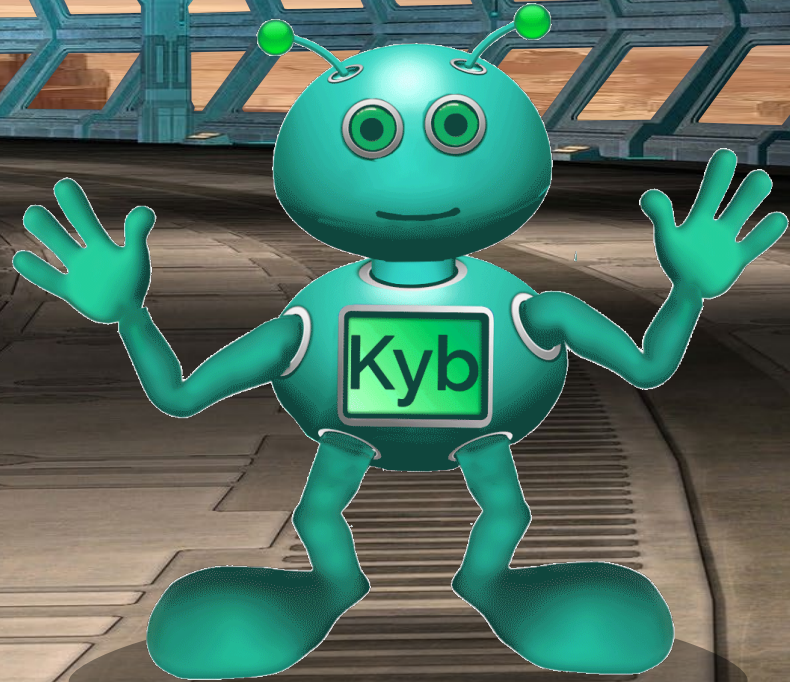
Dilithium



Signature method,  
replacement for RSA  
signatures

Kyber

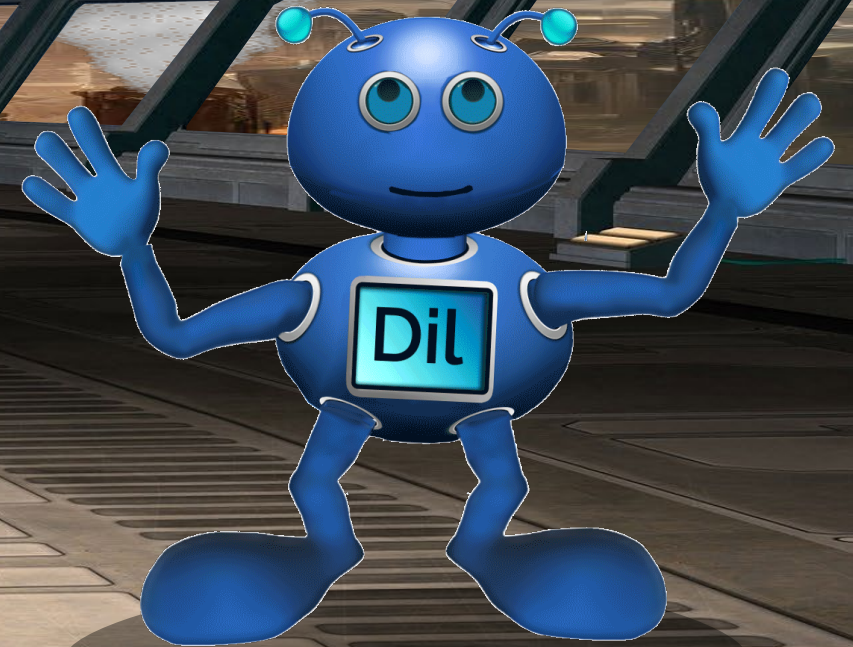
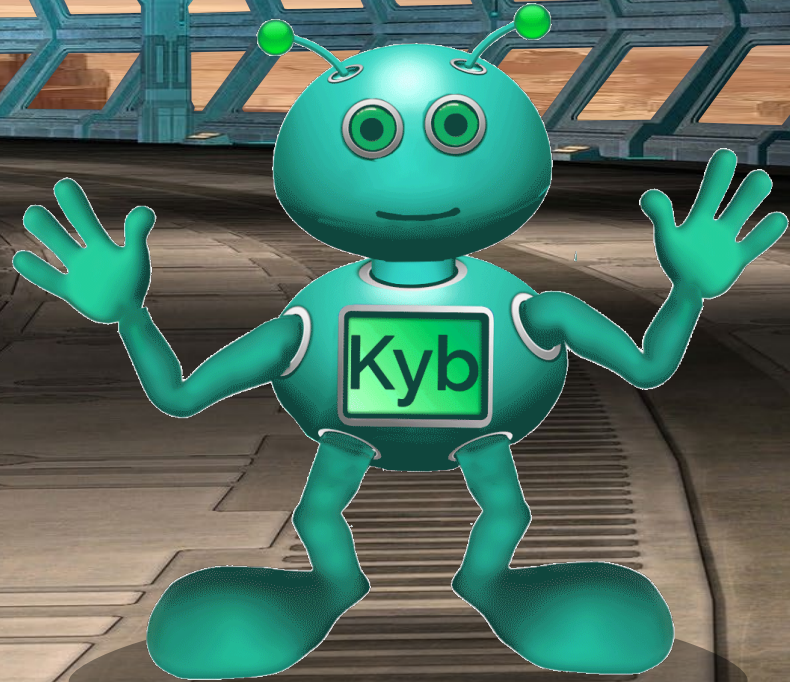
Dilithium



Both made in  
Germany

Kyber

Dilithium





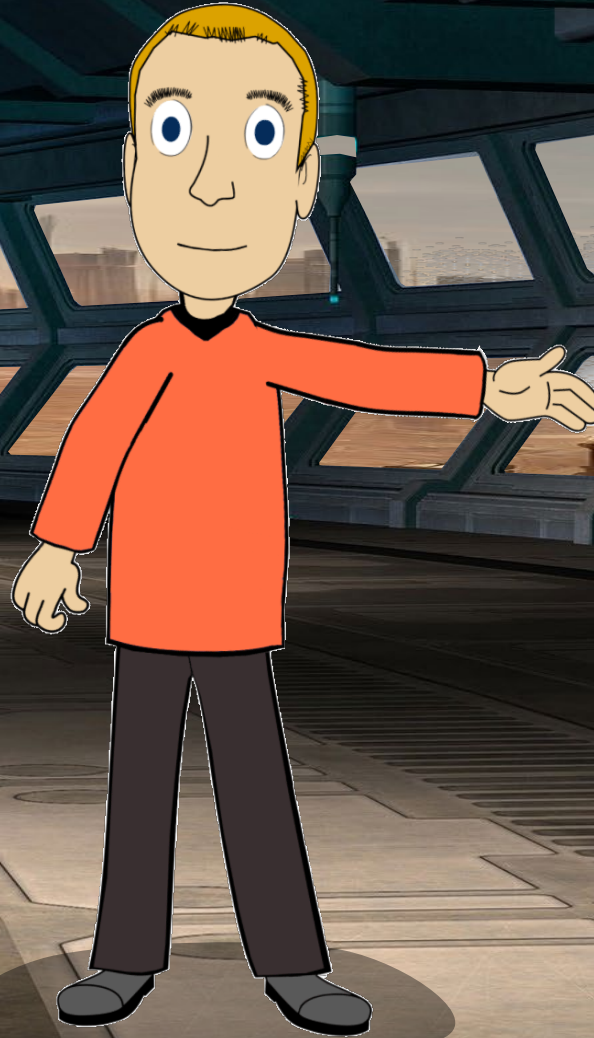
Eike Kiltz



Eike Kiltz



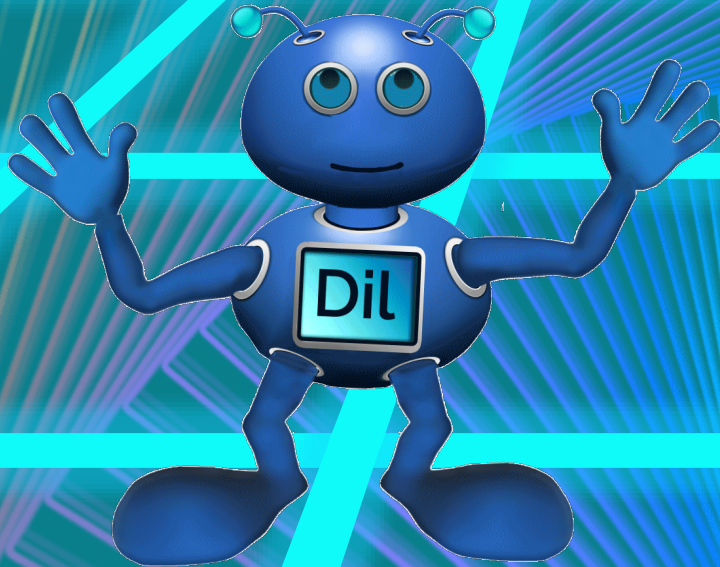
Peter Schwabe



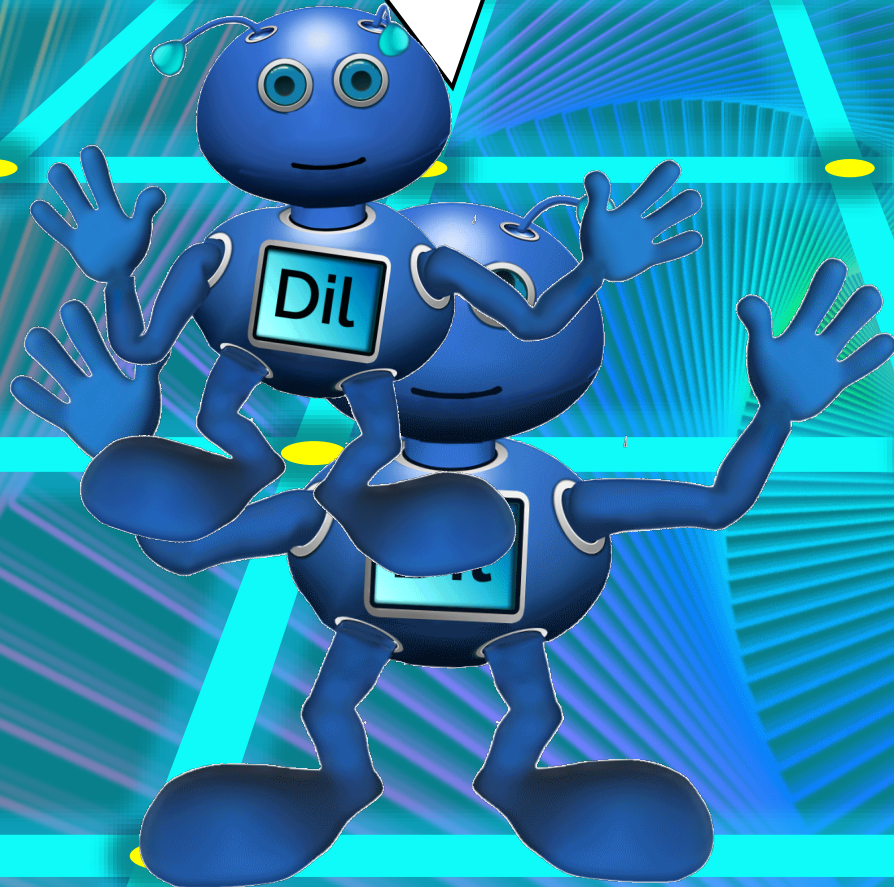
How the CRYSTALS-  
Kyber works is explained  
to us by Dil ...



I'm standing  
on a lattice.

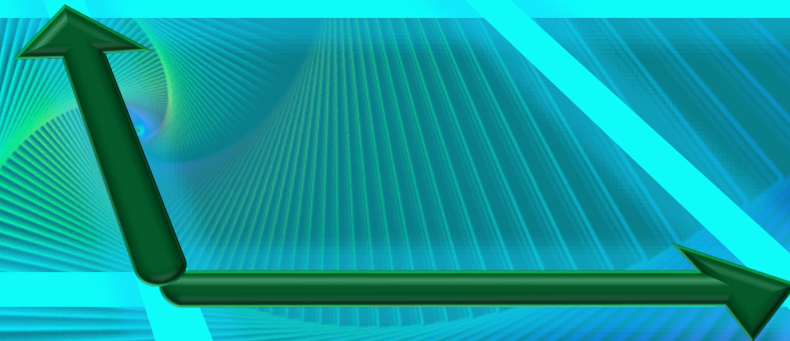


The intersections are called "lattice points".

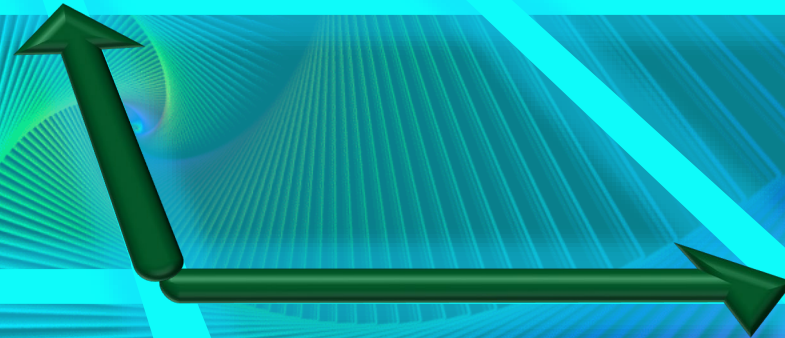
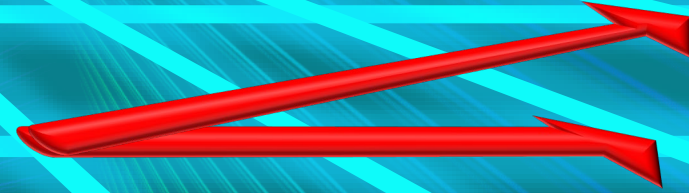
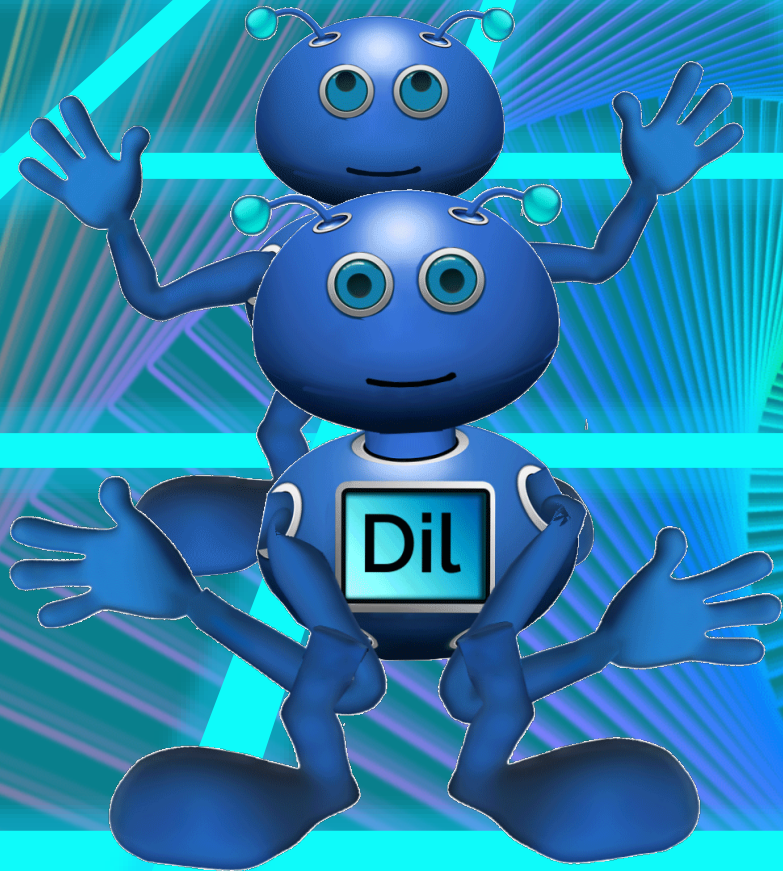


A lattice is defined  
with vectors.

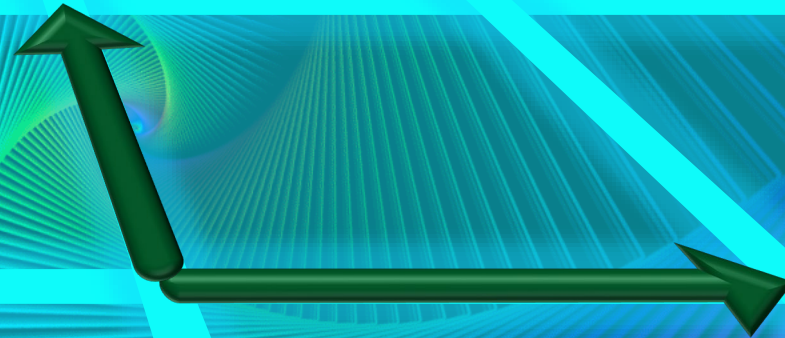
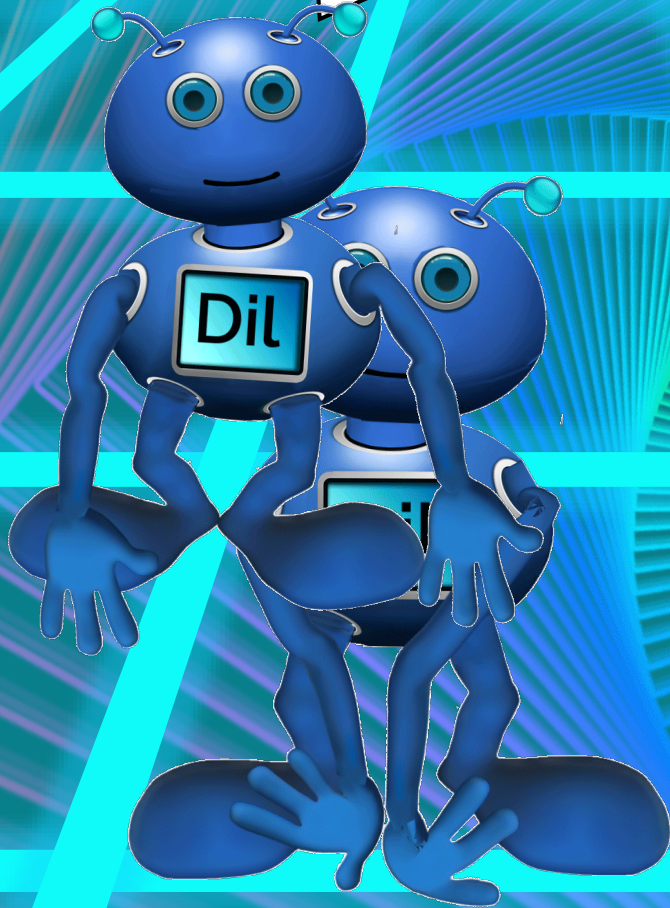
Reffered to  
as "base".



**Different bases may  
define the same lattice.**

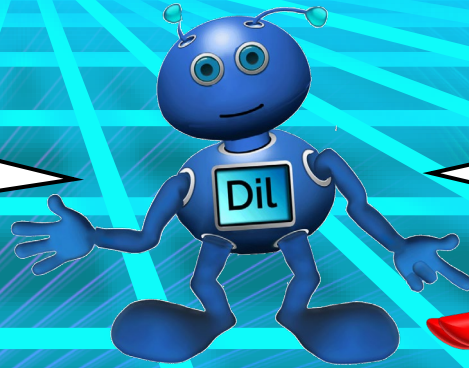


If the vectors are almost orthogonal, it's a "good base".

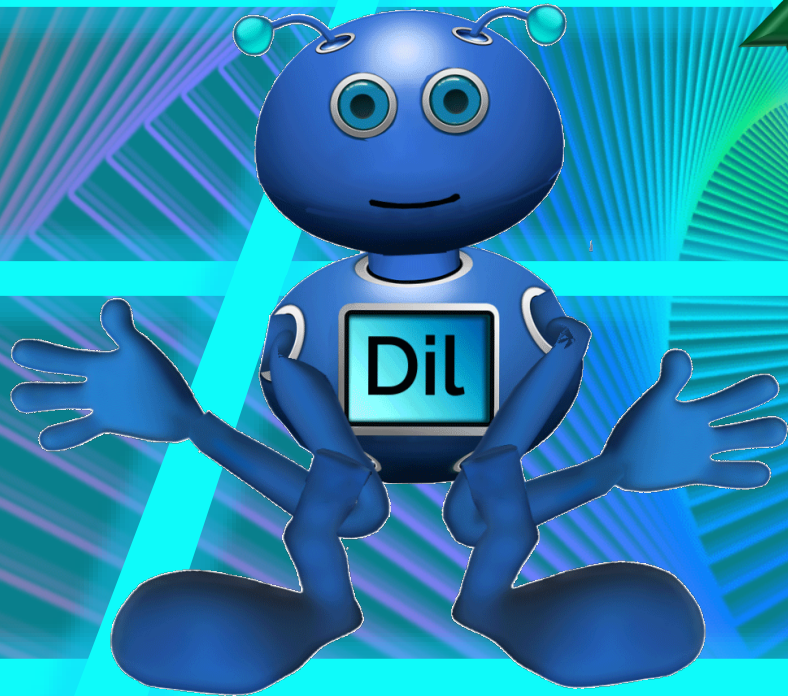




If the vectors are almost parallel, it's a "bad base".



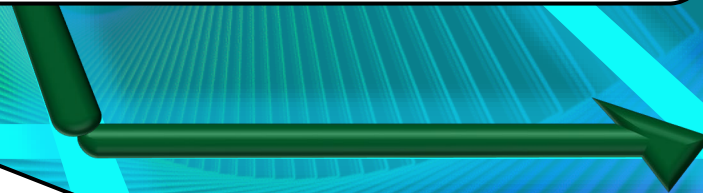
A bad base can be easily derived from a good one, but not the other way around.



**Closest-Vector Problem: Which lattice point is closest to a given point?**

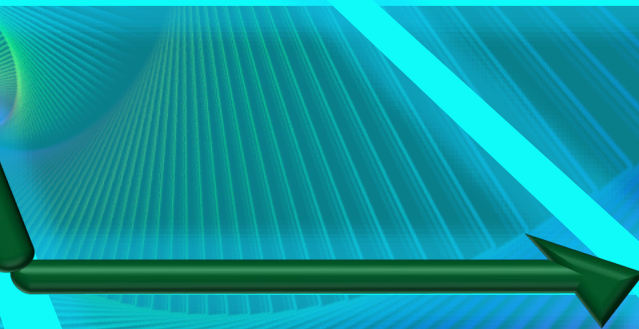
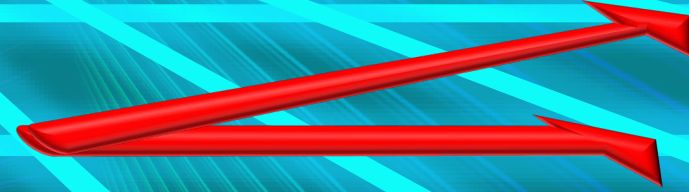
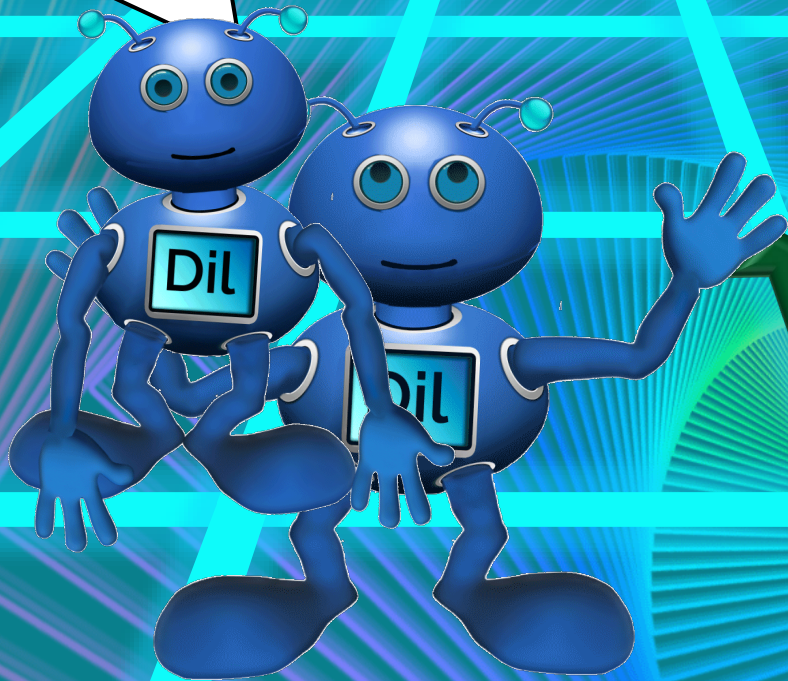


**Easy to answer in two-dimensional space.**

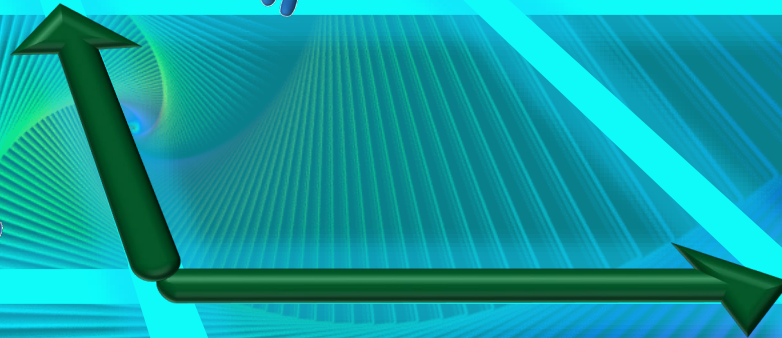
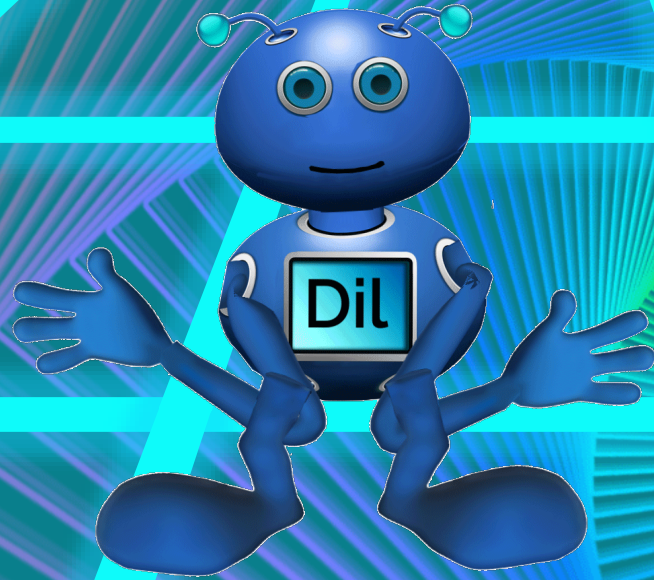


**In the 250-dimensional space:  
Easy to answer with a good base.  
Hard to answer with a bad base.**

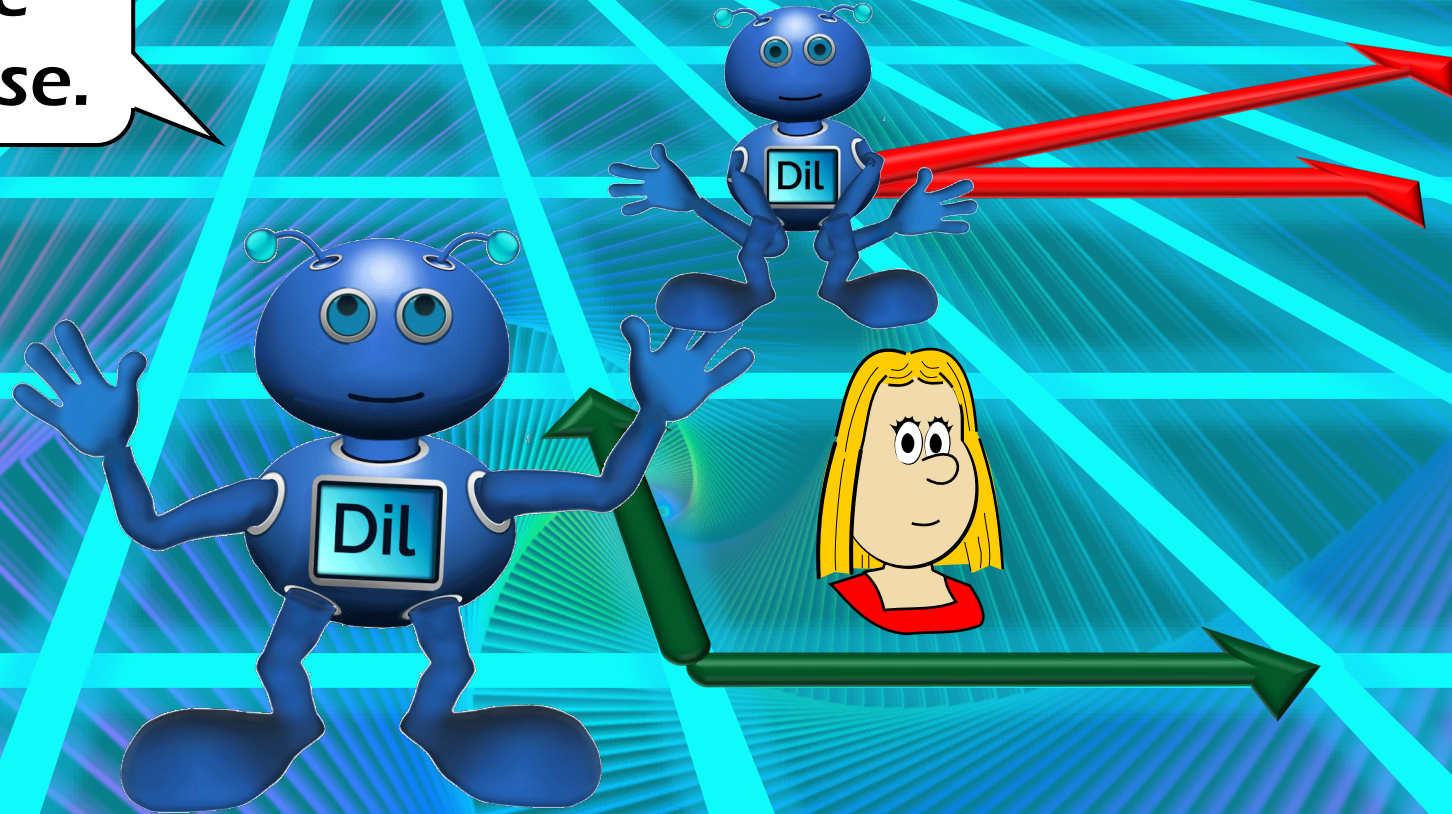
Lattice-based  
encryption

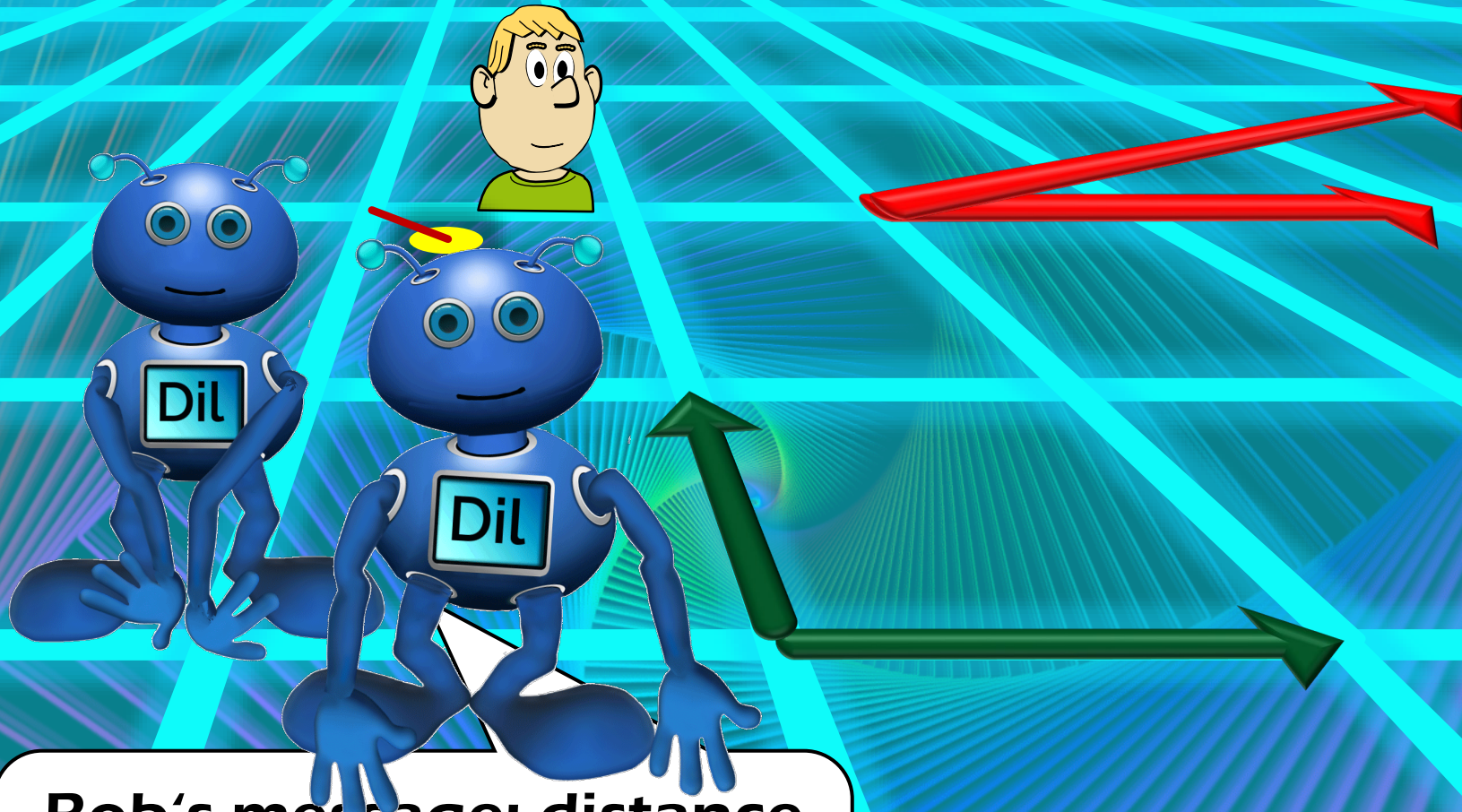


Alice's public key:  
a bad base.

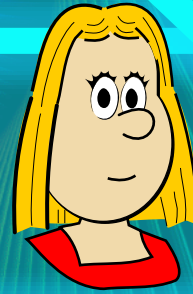


Alice's private  
key: a good base.

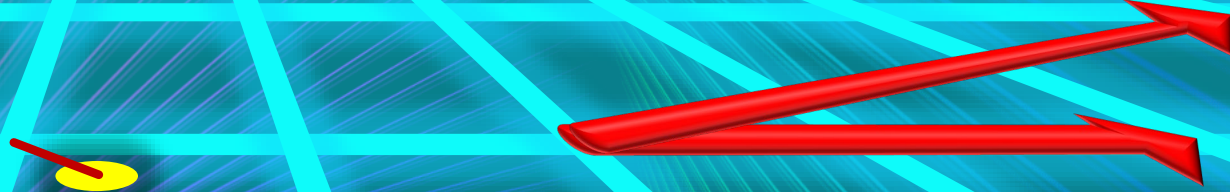


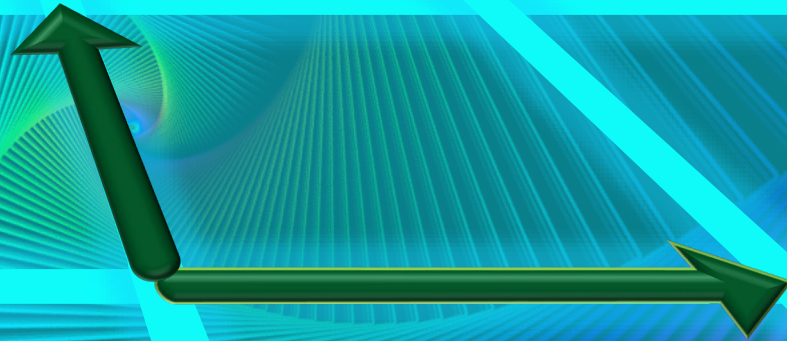
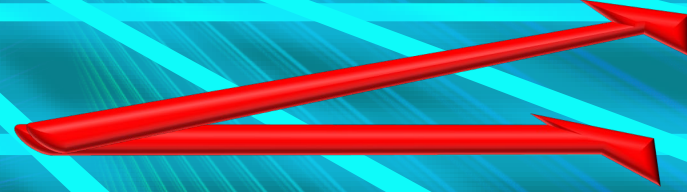


**Bob's message: distance  
between point and  
closest lattice point**



**Alice can derive  
closest lattice point  
with good base.**

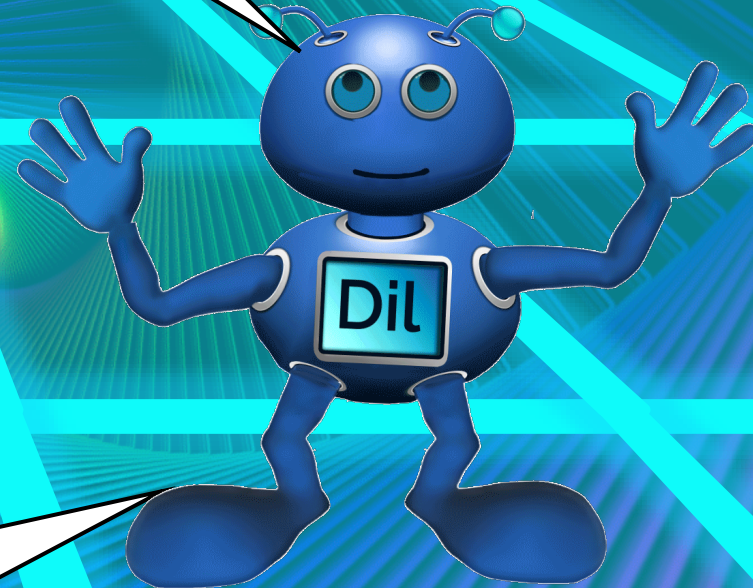
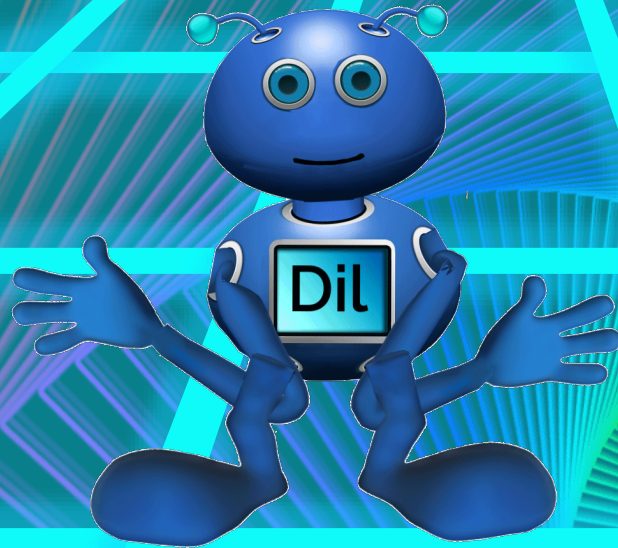




**Attacker can't derive closest lattice point with bad base.**

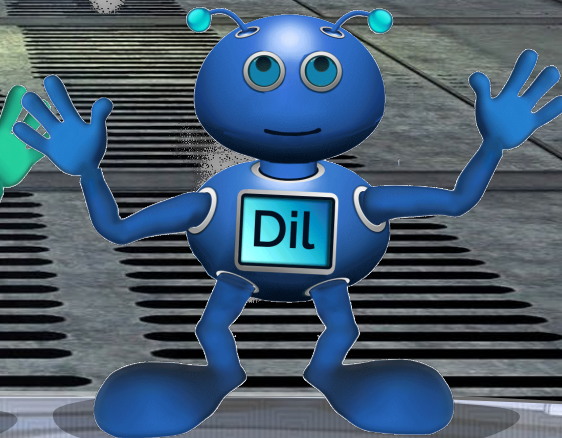
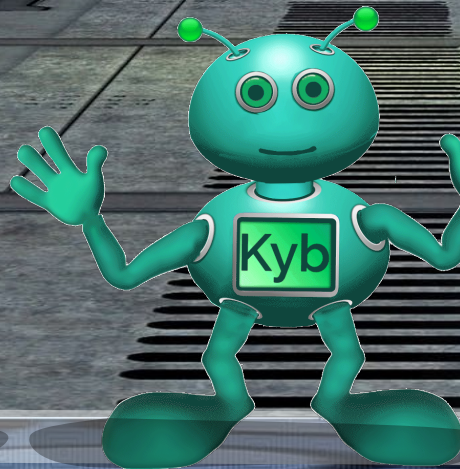
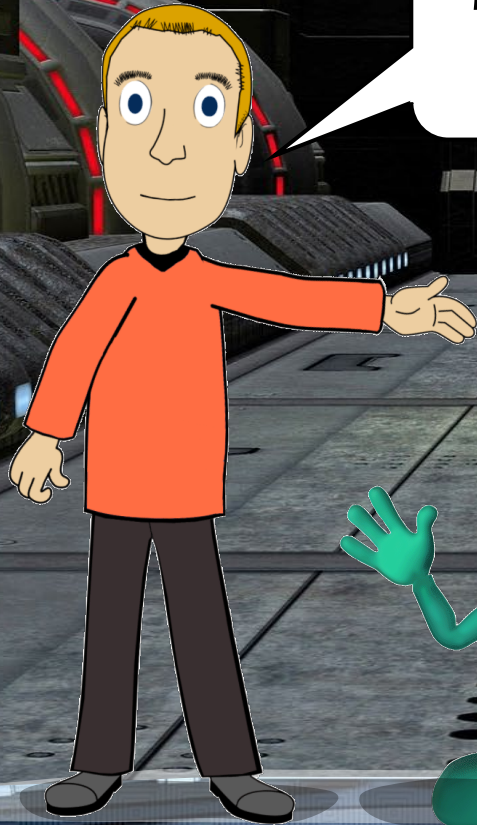


**This is how  
Kyber works.**

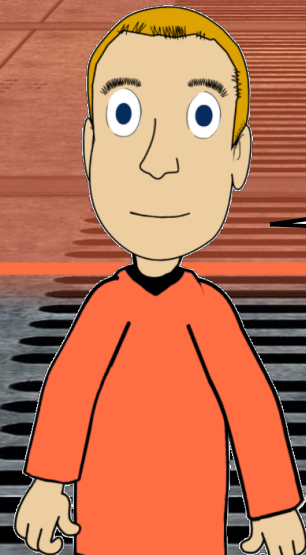


**Dilithium (signatures) is based  
on similar mechanisms.**

**Kyber and Dilithium are the winners of the year 2022!**



## Public-key lengths

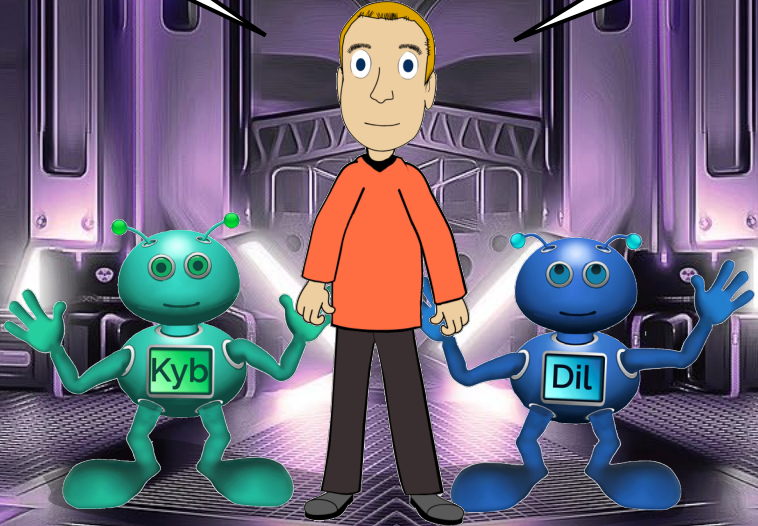


**The migration  
will not be trivial.**

# Conclusion

**Y2Q is approaching.**

**The means to beat quantum computers exist.**

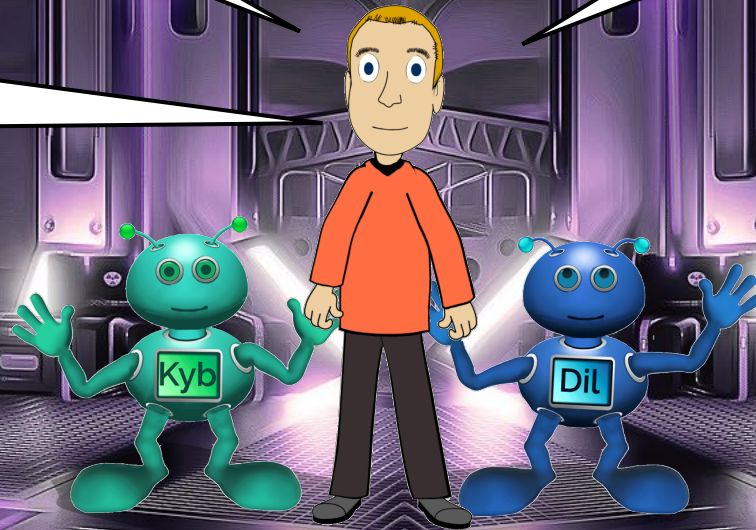


# Conclusion

**Standardisation is taking shape.**

**Migration will be the next step.**

**There's a lot to do, but it needs to be done.**





END